

# **OPRE 6V99.001**

Thursday: 1:00p.m.-3:45 p.m.

JSOM 12.214

1<sup>st</sup> class: August 21, 2017

## **STOCHASTIC DYNAMIC PROGRAMMING AND APPLICATIONS TO FINANCE AND OPERATIONS MANAGEMENT**

**Alain BENSOUSSAN**

### **1- MOTIVATION**

Stochastic Dynamic Programming is a general methodology which plays an essential role in many areas of economics and management science. It deals with decision making under uncertainty for dynamic systems, which is the situation appearing commonly in a lot of applications. Although this fact has been known for long time, it has become even more evident in view of the new problems which are presently at the forefront of research.

This is for instance true for mathematical finance, whose developments have motivated many innovative techniques related to SDP (Stochastic Dynamic Programming). The consumer-investor problem of Merton is the most famous one, but option pricing, credit risk, corporate finance, optimal compensation and many others have originated substantial new effort and interest in SDP related techniques.

In operations management, the application of SDP goes back to inventory theory, with the well known base stock and  $s, S$  policies. However, the structure of the supply chain has become very complex, and has justified new approaches, in particular the use of principal-agent methodology. For realistic situations, one has to consider dynamic systems, with moral hazard problems, asymmetric information, risk-sharing issues and incentive considerations.

A lot of interest is related to linking operations management with other management domains, like marketing and finance. The issue of pricing, traditionally a marketing question, is now considered together with capacity planning and inventory control. This new and very important topic is called revenue management. Dynamic models are essential, and lead to substantial extension of SDP.

Investment theory, growth of firms and real options are now connected and elicit new questions related to SDP methodology.

As soon as there are several decision makers, in competition, or contracting with each other, the issue of information comes naturally, with very delicate questions, for which intuition is of little help. A solid theory requiring sophisticated mathematical techniques is imperative. Even if mathematics is not the objective, it is indispensable to be comfortable with some of these techniques and concepts.

The case of a very large number of decision makers leads to the question of dynamic equilibrium, topics of strong interest in economics, but also for financial markets, and social networks. "Big data" has become the new framework, and involves new statistical problems, advanced simulation techniques and stochastic optimization with dynamic structure.

All these considerations have amply modified the type of background which is indispensable to perform relevant research in the current circumstances. They all have to do with stochastic dynamic programming and its evolution.

## **2- OBJECTIVE OF THE COURSE**

The objective of the course is to provide students with a solid background on SDP, with the core theory and its evolution to accommodate problems described in section 1. We shall also discuss the basic statistical methods used for model identification and calibration, for filtering and forecasting. We shall describe many models in finance and operations management, which have been studied and solved in the recent years. Not only, they represent operational examples of SDP, but also lead to important research topics of current interest. We shall describe the various principal-agent concepts for dynamic systems; make a presentation of the main ideas of differential games and its applications.

Risk management for dynamic systems will also be presented. A basic example in finance is the dynamic Markowitz problem, in which variance enters in the objective. These problems go beyond the traditional SDP approach. They necessitate a recent theory, called mean-field type control, which is in full expansion. Basic concepts and methodology will be presented.

We also will discuss the case of an infinite number of decision makers, which leads to equilibrium theory and mean field games.

Probably, not everything can be presented orally, but we shall provide students with lecture notes, as complete as possible, on all the topics described above. Our major objective is that the students get a good understanding on all the ideas and concepts, so they can become autonomous and work by themselves in more specific domains.

If PhD students are a major target of this course, we want to accommodate all master and graduate students, interested in extending their education and training or anticipating PhD studies in the future. This is why the course will be self-contained, and mathematical techniques will be fully explained before being used.

### **3- STUDENT WORK**

The students will be asked to show their understanding of the material by presenting written short syntheses of major questions. They will be involved in

projects related to extensions of models described in the course. They will have to perform numerical calculations and simulation using mat lab. PhD students already involved in research will be encouraged to take advantage of the course, and hopefully will develop their own models, using ideas related to the class.

Grading: Written and Oral Presentations, Participation 35

Mid Term Test 30

Assignments and Computer Projects 35

#### **4- SYLLABUS**

**This is an indicative program.** It will be adapted to the background of students.

The course will be self-contained. No prerequisites are required, but background in probability and optimization will facilitate the acquisition.

Class 1: Markov Chains

Class 2: Stochastic Optimal Control in Discrete Time

Class 3: Inventory Control without Set up Cost

Class 4: Optimal Stopping in Discrete Time

Class 5: Impulse Control in Discrete Time

Class 6: Inventory Control with Set up Cost

Class 7: Stochastic Processes: stochastic calculus, stochastic differential equations

Class 8: Continuous Time Inventory Control

Class 9: Inventory Control with Diffusion Demand

Class 10: Mean Reverting Inventory Control

Class 11: Stochastic Optimal Control: Stochastic Dynamic Programming

Class 12: Stochastic Maximum Principle, Backward Stochastic Differential Equations

Class 13: Stochastic Control Problems in Finance: Consumer-Investor problem, Entrepreneur decision making. Martingale method, duality

Class 14: Optimal Retirement: Optimal stopping, Variational Inequalities

Class 15: Principal Agent Problems: Risk sharing, implementing contracts. Contracting under hidden agent type

**Text Books:**

Alain Bensoussan, Dynamic Programming and Inventory Control, IOS Press, 2011

Alain Bensoussan, Estimation and Control of Random Dynamical Systems, Springer Verlag, to be published

These text books cover all the topics of the class, but are given only as references. PDF files will be provided. Also, Lecture Notes, following more strictly the contents of the lectures will be provided.

