Course Syllabus

PHYS 5301.501 Mathematical Methods in Physics Fall 2016

Professor Contact Information

Instructor:	Paul Mac Alevey
Office:	SLC 3.306
Phone:	extension 4634
E-mail:	paulmac@utdallas.edu I will send e-mail to UTD e-mail addresses only. Regulations prevent me from sending some information to any other e-mail address.
Mailbox:	in PHY 1.506

The 29 meetings of the class (inclusive of midterm tests) will be on Tuesdays and Thursdays in PHY 1.202 from 5:30 to 6:45

Course Pre-requisites, Co-requisites, and/or Other Restrictions:

There are no formal pre- or co-requisites for this class **but I assume that anyone enrolling in PHYS 5301 has taken courses on calculus of one and many (real) variables and a course about differential equations**.

An important pre-requisite is not a course but **is your time! You have to be willing to spend time with the material**. This course may be one of the more challenging in which you have enrolled. I also hope that it proves to be a rewarding course irrespective of your particular area of research. This course develops many mathematical techniques that you will come across while doing other physics courses or while doing research.

PHYS 5301 - Mathematical Methods of Physics I (3 semester hours) Vector analysis (and index notation); Cylindrical & Spherical coordinates; Sturm-Liouville theory; Legendre functions; integral transforms; differential equations (including Green functions). (3-0) Y

Office hours: Tuesday and Thursday 7:00 PM - 8:00 or by appointment.

I am quite happy to see you at these times. However, I do not use office hours just for the purposes of distributing hints to homework problems. If I am asked about a homework problem then my response will be to ask you what you have done. (Vague answers won't do. Expect me to ask you to write something.) I don't intend to do your homework for you. I expect that we will arrive at something to try. After talking to me, I expect that you have enough to go about solving the problem yourself!

TA: TBA Hours: TBA

Course Description:

The topics of this course are dictated by the needs of graduate physics courses that are in your future. For this reason, the course only fits together nicely when you are using this material in your future classes.

PHYS 5301 and PHYS 5302 are can be thought of as a single course.

The focus of this class is not on applications but on the development of tools with which to examine applications.

Chapter numbers are from the sixth edition of Arfken & Weber.

Ch 1 **Vector Analysis (Index notation – Cartesian tensors)**
We begin with some 'index notation' that will allow us to prove vector identities. In Math Methods II, this notation will be generalized to allow us to talk about tensors. We'll look at some vector analysis (Divergence theorem) and see that Gauss' Law suggests that we define Dirac's delta 'function'.

Ch 2 **Curved Coordinates and Tensors**
We just do a small part of this chapter. I just want to mention cylindrical and spherical coordinates. We'll need to write gradient in spherical coordinates. (Most of us are familiar with gradient in Cartesian coordinates only!)

Ch 10 **Sturm-Liouville Theory**

This is an important chapter. As undergraduates, we usually think of the elements of vector spaces as being directed line-segments. It is important to adjust our perspective so that we can discuss functions as elements of vector spaces. An immediate consequence is that functions can be written as a linear combination of simpler (basis) functions just as vectors are written as a linear combination of basis vectors. While this is an attractive analogy, it assumes that we can find a 'basis' of functions for our space in any given case. Finding a basis involves us in solving Eigenvalue problems and dealing with a restricted class or differential operators (Hermitian Operators). The technique of Separation of Variables (that you first saw as undergraduates) pre-supposes this background. This is an important technique in practice because many, second order, linear, PDEs can be solved in this way.

Ch 9 **Differential Equations**

Green Functions allow solutions of differential equations to be written as integrals. We'll begin with Green functions for ODEs (1 dimension) but Green functions are at their most useful when solving PDEs. If we are lucky, integrals can be done in closed form and so the solution can be written exactly. However, Green functions are quite useful even when the integrals can't be done. (For example, 'propagators' in Quantum mechanics are a kind of Green Function.)

Green Functions for differential operators L where $Ly \equiv \left(p(x)y'\right)' + q(x)y$ have especially nice properties. I will use section 10.5 to introduce these functions and continue them with section 9.7. Section 9.3 looks at separation of variables in PDE's. In separating PDEs in N variables, we produce N ODEs. Some of these ODEs are already familiar but some are new (and solutions of some of these ODEs are the 'special functions' in chapters 11 to 13).

Ch 12 **Legendre Functions**

These are one of the more common sets of 'Orthogonal Functions'. We will approach them through a generating function since they arise in this way in E&M. (You might remember the Legendre polynomials $P_0(x), P_1(x), P_2(x), \ldots$ appearing in your undergraduate courses.) The Legendre polynomials are member of a larger class of Legendre functions that can be used as a basis of a function space. These functions are an important part of the spherical harmonics that are the 'angular' part of several solutions of PDEs.

Return Ch 9 **Differential Equations**

Implementing the 'wavefront method' for solving PDEs gives very nice solutions in a couple of cases. The method gives us some feel for the structure of the solution space of a PDE and suggests a commonly-used classification scheme for PDEs.

Student Learning Outcomes:

- Students will use tensor algebra (Cartesian only) to generate a proof of a given vector or matrix identity
- Given a differential equation and sufficient initial data, students will construct a Green function with which to solve the differential equation
- Given an orthogonality integral for Legendre Polynomials (or Associated Legendre functions), students will expand functions (encountered in the specification of boundary values) on a basis of orthogonal functions. (Questions about this objective may occur in the context of boundary value problems.)

More broadly, I hope that students will;

- increase their familiarity with a variety of mathematical techniques
- increase in confidence in setting-up a problem so that it can be solved mathematically

It is worth remembering that you will stop taking classes at some stage and focus all of you energies on research. By then, you will need to be able to suggest questions to yourself as the subject of your work. This is only one class but nevertheless, it is worth remembering where you are headed. I don't expect you to come up with research questions during this course! I do expect that you begin to ask questions about the topic as you review/rewrite your lecture notes.[1] You might want to ask yourself some of the 'big picture' questions such as how a certain topic fits in with other things that you know; how the topic might be approached differently etc. You might want to ask me about such questions but I expect you to begin answering them yourself. In this spirit, we might add another aim of the course;

- Ability to consider open-ended questions.

I intend to present some material in the text in a slightly different fashion to the text and intend to include material that is not in the text when I think it relevant. (This extra material may be included in lectures, in hard-copy 'handouts' to the class or in postings to the eLearning site.) **It is your responsibility to use these sources in addition to the textbook**.

Required Textbooks and Materials:

This course uses *questions* adapted from *Mathematical Methods for Physicists* (6th edition; 2005) by G. Arfken and H. Weber (A&W). This book is available electronically on the library website, http://site.ebrary.com/lib/utdallas/docDetail.action?docID=10167050. **The adapted questions will be posted on eLearning and are the version to be done as homework**. This file of questions includes corrections of typos etc. in the questions in A&W.

A&W is a good reference book, a *fairly* good textbook and is commonly used for courses in math methods. ***I don't require you to buy this textbook.*** If you decide to get Arfken & Weber, use the 'Additional Readings' at the end of every chapter as other sources. Unfortunately, many students don't refer to other textbooks (or at least not early enough...)

The structure of my lectures is given in the schedule. No single textbook that I know does an excellent job with all of the topics. An additional constraint is that the department wants this course (and Math Methods II) to satisfy the mathematical needs other physics courses. So do I. **The intention is that 5301 and 5302 address material needed for other physics courses before it is needed in those courses.** ('Background' topics, that ultimately strengthen your use of mathematics as a tool to deal with physics, fit in wherever there is room for them.)

Your study habits **are more important than the textbook(s)** that you use. Because my lectures aren't drawn from a textbook, it is important that **you spend time working through the notes from lectures**. Read around the topic and add to your notes **as the course proceeds**. **Don't wait until tests loom before getting to grips with the subject**.

If you are looking for a cheap textbook then have a look at Mathematics of Classical and Quantum Physics by F.W. Byron & R Fuller. It is not a modern book but only costs $13 on Amazon.com

My favorite textbook (that I've used in my undergraduate class for years) is *Mathematical Methods in the Physical Sciences* by Mary L Boas*.* She covers much of the same ground that I describe in the schedule below. While the book is not (nominally) a graduate textbook, it is an excellent book to have. It might not go as far as you'd like but will certainly get you started.

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[1] I hope that the reference to 'reviewing/rewriting your notes didn't surprise you. By 'reviewing' I don't mean just checking that you actually have notes, handouts etc. I mean working through the algebra (pencil in hand) and following up on any questions that either I mention or that you think are worth considering.

Some students that have done this course liked *Mathematical Physics* by Sadri Hassani. (I thought that it took too long to get to topics of interest to your other classes.)

I am less familiar with *Mathematical Methods of Physics* by J. Matthews and R. L. Walker but this book was used for courses like Math Methods I and II. *Modern Mathematical Methods for Physicists and Engineers* by C.D. Cantrell looks to be a very nice book.

Looking beyond textbooks brings us to books of worked examples. *Vector Analysis* M. R. Spiegel and *Linear Algebra* by S. Lipschutz and M.Lipson (both about $14) are both Schaum Outlines. *Schaum's Outline of Fourier Analysis with Applications to Boundary Value Problems* by M. R. Spiegel. It also looks useful for this course and, like the other Schaum Outlines, has lots of examples.

Your undergraduate experience might have left you with the impression that there is a textbook out there that will resolve *all* your difficulties. There isn't. But there is a book out there that will resolve *some* of your difficulties. Please **identify something that you are content to use** (at least for now) **and get down to work quickly**.

Suggested Course Materials:

You should have a calculator, pencil and pen at all meetings of the class and at all tests. (The calculator need only be a 'scientific' one.)

Assignments & Academic Calendar:

Homework in this class takes the form of doing sets of questions. Most of the question numbers below are from Arfken and Weber's book though a couple of questions are my own. *I intend to send you an e-mail (on eLearning) on Tuesday evenings*. This will have the numbers of questions whose answers are **due at the <u>beginning of class</u>** on the **Thursday nine days later.** However, please don't wait for me to formally assign homework before you begin working. As soon as we finish section 1.4, consider problem 1.4.10 to have been assigned etc. (You will usually be right!).

The questions that are assigned for homework are those posted on eLearning. Several changes from questions in the book have been made.

The numbers of the homework questions are:

Section	Questions
1.4	10 (4) You don't have to use index notation for this question
1.5	12 (6), 13 (9) Use index notation for both questions
1.6	4 (6)
1.7	1(6), 5 (6)
1.8	3 (4), 8 (9) Use index notation for the first part
1.9	6 (9), 12 (6)
1.10	4 (6)
1.11	1 (9), 8 (12)
1.12	2 (6), 8 (4)
1.13	8 (9)
1.14	3 (9)
1.15	1 (6), 6 (4)
2.4	1(6), 3 (4), 8 (9)
2.5	10 (9)
10.1	2 (4), 4 (9), 8 (9)
10.2	2 (12)
10.4	5 (6)
10.5	2 (12), 1DGreen#4
	Extra Credit: Suppose that an infinite dimensional space of functions has the orthonormal basis $\{\phi_j(x)\}_1^\infty$. If $f(x)$ is an integrable function, show that if $(x) = a_1\phi_1 + a_2\phi_2 + \cdots$, that $a_j = \langle\phi_j\|f\rangle$. Hint; mimic the proof that we used in class for the infinite space V of vectors (such as $\vec{x}$). [Or generate another proof if you prefer]. Don't treat the space as a finite one and try and calculate the inner product directly.
9.3	4 (6)
9.7	1 (9), 3 (9)
12.1	3 (4)
12.2	3 (6)
12.3	5 (6), 12.3.potential of spherical shell
12.4	7 (12)
12.5	1 (12)
12.6	5 (6)
9.1	2.4.14 (24) [part a. Notice that the Laplacian of a *vector* is involved in part a. Part c. uses the 'Wavefront' method]

I intend to post solutions on the eLearning site after the due date. *Late homework is not accepted after solutions to questions have been posted on the eLearning site*.

To use eLearning, you have to have a login ID and password. The eLearning server is at https://elearning.utdallas.edu/webct/entryPage.dowebct. The solutions are protected with a password and can be opened with Acrobat 5 or later (available from http://www.adobe.com/products/acrobat/readstep2.html). The password on the PDFs is **methods**. It is intended that the solutions get you 'on a right track' that you can follow to produce a complete solution. *If you have any hard-copy solutions to problems from* A&W's *book, then I require that you destroy them or give them to me. Using solutions as a substitute for doing a problem yourself almost guarantees poor performance on exams*.

You will also need to **check the preferences used by your browser** from the 'check browser' link on the first page that you get after log-into eLearning. (eLearning uses pop-ups intensively. Use your internet options to make the site a 'trusted site'.)

A **tentative** schedule for the course is as follows;

Date		Aims	Meeting
Tuesday	Aug 23	Introduction	1
Thursday	Aug 25	Distribute first hand-out on 'index notation': Index notation representations of scalar product & matrix multiplication 1.3, summation convention Distribute second hand-out on 'index notation': Permutation symbol and representing the vector product 1.4,	2
Tuesday	Aug 30	Triple Product 1.5, Gradient 1.6 (Need to find $\frac{\partial r}{\partial x_j}$!!)	3
Thursday	Sept 1	Divergence 1.7 ex. divergence of $\vec{g} = \hat{r}g(r)$, Curl 1.8, Evett (1965)	4
Tuesday	Sept 6	worksheet in vector identities 1, 2 & 3	5
Thursday	Sept 8	worksheet identities #5 and #6 Unit coordinate vectors -Tangents to coordinate curves	6
Tuesday	Sept 13	Cylindrical coordinates 2.4 & Spherical coordinates 2.5, Divergence theorem 1.11	7
Thursday	Sept 15	Stokes' theorem 1.12, Gauss' Law from Coulomb's Law 1.14, Solid angle (1.14 and 2.5)	8
Tuesday	Sept 20	Dirac's Delta function 1.15	9
Thursday	Sept 22	Showing $\delta(x-a) = \delta(a-x)$. Show $\delta(ax) = \frac{1}{\lvert a \rvert}\delta(x)$. SPLIT INTO A>0 AN A<0 TO OUTSET Chapter 10 - Sturm-Liouville Theory/Hilbert space CHECKTHE HO!!	10
Tuesday	Sept 27	Inner Product "Closure relation" for the Delta 'function' is from 1.15	11
Thursday	Sept 29	**Midterm I**	12
Tuesday	Oct 4	Completeness & Parseval's Equality/Bessel's Inequality 10.4, Decomposition in an infinite dimensional space requires orthogonality of basis	13
Thursday	Oct 6	Eigenvalue problems as a source of orthogonal functions	14

Tuesday	Oct 11	Condition for Orthogonality 10.1 Getting Lagrange's Identity & Hermitian Operators 10.2	15
Thursday	Oct 13	BCs needed to make operators Hermitian Self-Adjoint form of 2nd order ODEs 10.1	16
Tuesday	Oct 18	One Dimensional Green Functions (in section 10.5, pages 663 – 670)	17
Thursday	Oct 20	Calculating a one Dimensional Green Function , Green functions for PDEs 9.7	18
Tuesday	Oct 25	First way to find Green function for Laplacian Operator Second way to find Green function for Laplacian Operator, Divergence and curl in 3D orthogonal coordinates	19
Thursday	Oct 27	Finding a Green function for the Helmholtz Operator Eigenfunction expansion of the Green Function for the Helmholtz Operator (beginning of 10.5) & Symmetry of the Green Function. Separation of Variables 9.3	20
Tuesday	Nov 1	Generating Function for Legendre Polynomials 12.1 Special values of Legendre Polynomials,	21
Thursday	Nov 3	Recurrence relations for Legendre 12.2 (plus the use of recurrence to show that these Legendre functions [that begin $P_0(x) = 1, P_1(x) = x, \cdots$] are actually polynomials.)	22
Tuesday	Nov 8	Second order ODE for Legendre Polynomials from recurrence relations Orthogonality of Legendre Polynomials 12.3	23
Thursday	Nov 10	**Midterm II**	24
Tuesday	Nov 15	Normalizing the Legendre Polynomials 12.3 Repeated differentiation of products	25
Thursday	Nov 17	Rodrigues formula 12.4, ODE for Associated Legendre Functions 12.5	26
Tuesday	Nov 22	*University closed. Fall Break!!*	
Thursday	Nov 24	*Happy **Thanksgiving**!*	
Tuesday	Nov 29	Parity of Associated Legendre functions Choices for orthogonality of Associated Legendre Functions 12.5	27
Thursday	Dec 1	Choices I & II for Orthogonality of Associated Legendre Functions 12.5 Normalization of Associated Legendre Functions 12.5	28
Tuesday	Dec 6	Spherical Harmonics 12.6 Partial Differential Equations (9.1) - 'Wavefront' method,	29
TBA		**Comprehensive final - TBA**	
		If we have time then I'll include something about; Classification and solutions of second order PDEs (9.1) Examples of hyperbolic and parabolic PDEs Fourier Transforms 15.2, Solution of PDEs with Transforms 15.4 Transfer function 15.7, Convolution Theorem 15.5 Parseval's relation for Fourier Transforms in 15.5	

The University arranges the final exam. (They haven't scheduled it yet). Please check the UTD web page to check the scheduled time just before this exam.
http://www.utdallas.edu/student/registrar/finals/

- I do not intend to cover all sections in the text

- I do not intend to follow the order in which the material is presented in the text

- I intend to present some material in the text in a slightly different fashion from the text. Please take good notes!

- **Expect that <u>my</u> test** dates won't change. Content of tests may change but will not include material in chapters/sections that have not been treated in class.

Grading Policy:

The university calculates a GPA for graduate students based on the grades A, A-, B+, B, B-, C, C-, F, etc. (If you are a PhD student in the Physics dept. and want to make a research proposal, then you need a GPA that corresponds to B+ in your courses at UTD. You don't want to get lower than a B (that is B-, C, ….). Ask Dr. Anderson, Dr. Lee or Barbara Burbey for more details.) I intend to use a grade scale as follows. If *x* is a score then,

$x \geq 90$	A
$90 > x \geq 85$	A-
$85 > x \geq 75$	B+
$75 > x \geq 65$	B
$65 > x \geq 60$	B-
$60 > x \geq 55$	C+
$55 > x \geq 45$	C
$45 > x$	F

Weighting:	Homework	18%
	Midterm tests	26% each
	Final Exam	30%

I do not intend to use a curve in my grading of individual tests. A grade of X (incomplete) is awarded if an unforeseen, non-academic emergency prevents a student from completing the work in a course. If a student wants to discontinue the course because a poor grade is expected, it is nearly always more appropriate for the student to <u>withdraw</u> from the course and re-register in another semester. If an incomplete is given, the course must be completed within eight weeks of the first class day of the next long semester.

My tests are 'closed book' and 'closed notes'. **I tend to embed reference material and some long equations in my tests**. I have found that the main difficulty with tests is not with remembering equations but in knowing how to use them. **All books, notes, backpacks, cell phones etc. are to be placed by the sides of the room during a test**. (By the way, don't spend too long erasing mistakes when writing answers to **test** questions. Begin again and **label the correct version** so that I can find it. Versions that you can't get to work may tell me something.)

Use of <u>scientific</u> calculators is allowed on tests. However, graphing and programmable calculators are not allowed. None of the test questions that I ask will involve lots of number crunching. **Valid UT-D student cards must be available if requested during tests**. (You can get one made at the info depot in the student union building; SU 2.204.)

Missed tests can only be made up in the case of documented, extenuating circumstances. Such circumstances include medical emergencies and work-related travel that cannot be re-scheduled.

Course & Instructor Policies

Doing homework is an important part of the learning process. **Feel free to form study groups etc**. However, it is important to **hand in work that is your own**. When writing an answer to a question, it is <u>important to write comments that explain both what you are trying to do</u> and how you are trying to achieve it. In addition to the homework problems that are handed in for grading, I may suggest that you <u>work problems other than homework problems</u>. These are worth looking at as they improve your ability to solve problems.

In addition to helping you become familiar with the material, the homework will include problems that have longer solutions than problems on a test. <u>Begin your homework when it is assigned</u>; many problems are too difficult for a last-minute effort. When grading your work, the grader will be trying to understand your reasoning. Help him/her by saying what you are trying to do! Homework with no comments or partly scratched out answers don't help you show this. For grading, present neat versions of your solutions to the TA. Answers that are indecipherable will not attract much credit. If needed, **I may ask any student to explain their work to me.**

Dishonesty:

I would like to emphasize a point about the use of secondary sources etc. I do not object to people discussing problems that they have already attempted**. I do not object to the use of any other textbooks that you come across.** I object strongly to any verbatim, unacknowledged work done by anyone other than you and presented as part of your work**. (This includes any passages from textbooks, any solutions that you come across in hard copy or on eLearning etc. It also includes work produced by any other member of the class [past or present]). Every student in the course agrees to this limitation. Further, all students agree to tell me the source of any solution to problem assigned in this course that they know about. No materials posted on the eLearning site become the property of the student. At the conclusion of the course, all students undertake to keep all course materials for their exclusive use. Any distribution of course materials to third parties constitutes academic dishonesty and will be reported to the Dean of Students**

In order to further the objective of eliminating scholastic dishonesty, the University has a policy on scholastic dishonesty. This policy is clearly articulated in Subchapter F section 49.36 of the policy on student discipline & conduct adopted by the University and used in this course. A link to chapter 49 is at http://www.utdallas.edu/judicialaffairs/UTDJudicialAffairs-HOPV.html Students enrolling in the course are bound by this policy and are encouraged to read it. Any questions about this policy can be asked of the Dean of Students. **Any suspected cases of scholastic dishonesty will be passed along to the Dean of Students**.

Students are welcome to ask questions of my TA or me about homework problems. However, I do not authorize these students to communicate such discussions to other students. These other students are welcome to ask me questions too.

The eLearning site contains postings exclusively for the use of the person with the privilege accessing the site. Materials on this site form another secondary source that is intended to help students in my class during the semester that the posting is made. No materials posted on the ELearning site become the property of a student. **Students acknowledge that distribution/transmission of any posting made on the eLearning site constitutes scholastic dishonesty**. (See parts (d) 1 and (d) 5 of section 49.36 of the policy on student discipline & conduct.)

The question about postings on the eLearning site can be extended. I will treat in the same way any pre-existing solution to a problem assigned as homework in a previous semester, a solution to a problem asked on a test, or any problem from the book. **As soon as any student in this course comes across any kind of pre-existing solution, that student must inform me of its existence and source**. To do otherwise is to aid copying. (See part d (1) of section 49.36.) In order to maintain privacy, I can be contacted by e-mail if desired.

At the conclusion of the course, all students undertake to keep **all** course materials (posted solutions, graded homework etc.) for their exclusive use.

A note about missing classes

First of all, please try not to! **If something arises that prevents you from attending class, please inform me as to why by e-mail**. Not everything that we do in class is covered in any single textbook. If there were an ideal textbook for us then there would be little or no need for classes. An ideal textbook does not exist. By missing class, you will miss either something not covered by the book that you are reading, or you will miss 'intermediate steps' in an author's argument that will help you follow along. You also pass up the opportunity to ask questions of your own and miss out on hearing the questions of others. (This latter point is significant. Other students may ask questions that haven't occurred to you yet and hence develop your understanding of the subject.) If you **have** to miss class for some reason then it is your responsibility to get class notes or handouts given in class. (I'm not keeping tabs on your attendance and leave some of the responsibility to you.) Please do this quickly after your absence. In order to understand the next lecture given, you will need to have obtained and worked through any notes etc. from the previous lecture. I give lectures from 'outline notes' that are probably not what you want to read. If you miss a lecture then your best source of class notes is another student who wrote down exactly what we actually did. I return graded homework and tests primarily in class. Again, you'll miss this if you are absent from class. After I have tried to return the graded work to you a class from which you were absent, the responsibility for getting it from me becomes yours.

Comet Creed

This creed was voted on by the UT Dallas student body in 2014. It is a standard that Comets choose to live by and encourage others to do the same:

"As a Comet, I pledge honesty, integrity, and service in all that I do."

UT Dallas Syllabus Policies and Procedures

The information contained in the following link constitutes the University's policies and procedures segment of the course syllabus.

Please go to http://go.utdallas.edu/syllabus-policies for these policies.

The descriptions and timelines contained in this syllabus are subject to change at the discretion of the Professor.