University of Texas at Dallas

Course Syllabus

Math 3351.501: Advanced Calculus
Spring 2023

Time and Location:	Tues & Thurs 11:30 am – 12:45pm	
Location:	SCI 2.210	
Instructor:	Wieslaw Krawcewicz	Professor
Contact Information:		
• Office:	FO 2.602F	
• Phone:	214-708-3076 (private)	for text only
• Email:	wieslaw@utdallas.edu	
Office Hours:	Tuesdays -Thursdays	4:00 pm - 5:00 pm

Course Description:

MATH 3351 - Advanced Calculus (3 semester credit hours): The course covers the interplay of linear algebra, higher dimensional calculus, and geometry. Topics include vectors, coordinate systems, the elementary topology of Euclidean spaces and surfaces, the derivative as a linear map, the gradient, multivariate optimization, vector fields, vector differential operators, multiple integrals, General Stokes Theorem, and differential forms. Applications are given to geometry, science, and engineering. Basic topological intuition is developed.

Course Pre-requisites, Co-requisites:

A grade of at least a C- in either MATH 2415 or MATH 2419 or equivalent, and a grade of at least a C- in MATH 2418 or equivalent.

Textbooks:

1. C.H. Edwards, Jr., *Advanced Calculus of Several Variables*, Dover Publications 1973, New York (recommended).
2. Peter D. Lax, Maria Shea Terrell, *Multivariable Calculus with Applications*, Undergraduate Texts in Mathematics, Springer 2017 (optional).
3. David M. Bressoud, *Second Year Calculus*, Undergraduate Texts in Mathematics, Springer 1991 (optional).

Homework Assignments:

There will be about 10 mandatory graded assignments. Assignments will contribute 20% to your final grade. The homework assignments will be published on eLearning website and the completed homework should be submitted directly to the instructor on in class on the due date.

Grading Policy:

• Homework assignments:	20%
• Midterm Exam 1:	25%
• Midterm Exam 2:	25%
• Final Exam:	30%
• Total:	100%

Grade Scale:

A+	[95…100]	A	[86…94]	A-	[80…85]
B+	[75…79]	B	[70…74]	B-	[65…69]
C+	[60…64]	C	[55…59]	C-	[50…54]
D+	[46…49]	D	[44…45]	D-	[42…43]
F	[0…41]				

Midterm Exams:

All midterm exams will be held on Saturdays in a reserved classroom SCI 2.210 at 2:00 pm.

	Date	**Time**	**Location**
Midterm Exam 1:	March 11, 2023	2:00 - 4:00 pm	in SCI 2.210
Midterm Exam 2:	April 8, 2023	2:00 - 4:00 pm	SCI 2.210

Class Notes

Class notes will be recorded and made available to all students registered in this class.

Student Learning Objectives:

(a) Students will learn the following concepts:
- the concept of differentiability for functions of multiple variables,
- derivative as a linear operator,
- surfaces and tangent planes,
- the concept of divergence and curl,
- higher derivatives of functions of multiple variables and their
- local and global extrema, Lagrange Multipliers
- integral of function of several variables (multiple integrals),
- iterated integrals, volumes in n-dimensional space,
- change of variables,
- line integrals,
- surface integrals,
- differential forms and Stokes theorem;
- calculus of variations.

(b) Students will learn the following Theorems: • Sufficient Conditions for Differentiability for functions of several variables, • Chain Rule, • Inverse and Implicit Function theorems, • Lagrange Multiplier, • Green's Formula, • Divergence and Stoke's Theorems.

(c) Students will show ability to apply the learned concepts to the real world problems: • motion in space, • description of planetary motions, • surface areas and volumes of objects in multidimensional spaces, • optimization problems.

Detailed Description Tentative Schedule of the Course

Week 1. Review of Linear Algebra in Euclidean space $\mathbb{R}^n$: linear combinations of vectors, linear independence, dot-product and Euclidean norm in $\mathbb{R}^n$, orthogonal projections, orthonormal sets in $\mathbb{R}^n$, linear transformations, matrices, norm of a matrix, determinant, finding inverse matrices, and volumes of parallelepiped, (Textbook: Chapter 1, sections 1.1 - 1.6, Chapter 5, section 5.3).

Weeks 2-3. Geometry in Euclidean space: line, planes and hyperplanes, cross-product, polar and spherical coordinates, equations of conic surfaces, parametric curves and modeling motion in $\mathbb{R}^n$. Topology of the Euclidean space $\mathbb{R}^n$: open balls, open sets, closed sets,

boundary, interior and closure of a set, sequences, convergence of sequences, compact sets, connected sets Functions of multiple variables: limits of functions ($\varepsilon - \delta$-definition) and their properties, examples of limits, continuity and uniform continuity of functions, Extreme Value Theorem, properties of elementary functions, using other coordinate systems for expressing functions in $\mathbb{R}^2$ and $\mathbb{R}^3$, graph of a function. Examples. (Textbook: Chapter 1, sections 1.7- 1.8).

Week 4. Differentiation of functions of multiple variables: differentiability of a function, derivative as linear transformation, partial derivatives and the matrix representation of a derivative, directional derivatives, continuity of differentiable functions, Sufficient Differentiability Condition, Jacobian and its interpretation, gradient of a function, notion of a continuously differentiable function. Examples. (Textbook: Chapter 2, section 2.1-section 2.2).

Week 5. Properties of Derivatives: surface as a graph of afunction, tangent space to a surface, gradient and level sets, Chain Rule (Textbook 1: Chapter 2, section 2.3). Higher derivatives of functions of several variables: higher partial derivatives, operator interpretation of higher derivatives, Clairaut's Theorem. Extrema of a function, Critical and regular points. . First Derivative Test, Hessian matrix and Second Derivative Test, extrema on the level set and Lagrange Multiplier Theorem. Taylor's Formula. (Textbook: Chapter 2, sections 2.4-2.5).

Week 6. Mean Value Theorem Inverse Function Theorem, Implicit Function Theorem. Manifolds in $\mathbb{R}^n$. (Textbook, Chapter 3, sections 3.1-3.5.

Week 7. Integration: volume as integral, contented sets (regular domains), upper and lower integrals, integrability of function, properties of multiple integrals on contented sets (and regular domains). (Textbook: Chapter 4, sections 4.1 - 4.3).

Week 8. Multiple integral as iterated integrals, Fubini's theorem, and change of variables in multiple integrals. (Textbook: Chapter 4, sections 4.4 - 4.5).

Week 9. Improper integrals and absolutely integrable functions (Textbook: Chapter 4, section 4.6).

Week 10. Line integrals: line integral of the first type (of scalar functions over a curve), average value of a function over a curve, line integral of vector-valued functions, conservative vector fields, Fundamental Theorem of Line Integrals, Green's formula. (Textbook: Chapter 5, sections 5.1 - 5.2).

Week 11. Surfaces and surface integrals. (Textbook 1: Chapter 5, sections 5.4).

Week 12. Classical theorems of vector analysis: Divergence Theorem in $\mathbb{R}^3$ and Stokes Theorem in $\mathbb{R}^3$. (Textbook: Chapter 5, sections 5.7 - 5.8).

Week 13. Closed and exact forms. (Textbook: Chapter 5, sections 5.8).

Week 14. Calculus of variations (Textbook: Chapter 6).

Additional Information

Classroom Safety and COVID-19

To help preserve the University's in-person learning environment, UT Dallas recommends the following:

• **All Comets are strongly encouraged to wear face coverings indoors and practice social distancing regardless of vaccination status.** Please adhere to the University's CDC Updated Guidelines issued on July 30, 2021. Please note this represents a change in the campus guidance issued on May 20, 2021.

• **Accommodations for students who must isolate or quarantine due to COVID.** To keep the UT Dallas community as safe as possible, the University requires students who test positive for COVID-19 or who are close contacts as determined by the campus contact tracing program to isolate or quarantine as applicable. Video recordings of lectures will be made available for those students.

• **Verifying COVID-19 isolations or quarantines.** Students need to self-report COVID-19 positive results or exposures via an online form so that university campus tracers can verify, record, and take necessary campus precautions. Students should not attend class until cleared by campus tracers.

• **Vaccinations are widely available, free and not billed to health insurance.** The vaccine will help protect against the transmission of the virus to others and reduce serious symptoms in those who are vaccinated. You are encouraged to get a COVID-19 vaccine and register your vaccination status through the voluntary vaccine report form.

• **Proactive community testing** remains an important part of the university's efforts to protect our community. Tests are fast and free. Please check the Comets United webpage for additional information.

Student safety is an important part of the UT Dallas' efforts to protect our community. All students will adhere to the Comet Commitment. Unvaccinated Comets will be expected to complete the mandatory Required Daily Health Screening. Those students who do not comply will be referred to the Office of Community Standards and Conduct for disciplinary action under the Student Code of Conduct – UTSP5003.

Comet Creed

This creed was voted on by the UT Dallas student body in 2014. It is a standard that Comets choose to live by and encourage others to do the same: *"As a Comet, I pledge honesty, integrity, and service in all that I do."*

UT Dallas Syllabus Policies and Procedures

• The information contained in the following link constitutes the University's policies and procedures segment of the course syllabus: http://go.utdallas.edu/syllabus-policies

• *These descriptions and timelines are subject to change at the discretion of the Professor.*